NIAGARA ACADEMY
ADVANCED FUNCTIONS, GRADE 12, UNIVERSITY PREPARATION
COURSE OUTLINE

COURSE CODE:	MHF4U
DEVELOPED BY:	R. Cooper, September, 2007
REVISED BY:	R. Cooper (2009), M. Wilson (2013), T. Bozza (2014) S. Davis (2015, 2016), J. Pauls (2017, 2018, 2019)
DEVELOPED FROM:	The Ontario Curriculum Grades 11 and 12, Mathematics, 2007, http://www.edu.gov.on.ca/eng/curriculum/secondary/math1112currb.pdf
PREREQUISITE:	Functions, Grade 11, University Preparation or Mathematics for College Technology, Grade 12, College Preparation
COURSE DURATION:	110 hours
COURSE VALUE:	1.0 credits

COURSE DESCRIPTION AND RATIONALE

This course extends students' experience with functions. Students will investigate the properties of polynomial, rational, logarithmic, and trigonometric functions; develop techniques for combining functions; broaden their understanding of rates of change; and develop facility in applying these concepts and skills. Students will also refine their use of the mathematical processes necessary for success in senior mathematics. This course is intended both for students taking the Calculus and Vectors course as a prerequisite for a university program and for those wishing to consolidate their understanding of mathematics before proceeding to any one of a variety of university programs.

OVERALL CURRICULUM EXPECTATIONS

Exponential and Logarithmic Functions:

By the end of this course, students will:

- demonstrate an understanding of the relationship between exponential expressions and logarithmic expressions, evaluate logarithms, and apply the laws of logarithms to simplify numeric expressions;
- identify and describe some key features of the graphs of logarithmic functions, make connections among the numeric, graphical, and algebraic representations of logarithmic functions, and solve related problems graphically;
- solve exponential and simple logarithmic equations in one variable algebraically, including those in problems arising from real-world applications.

Polynomial and Rational Functions:

By the end of this course, students will:

- identify and describe some key features of polynomial functions, and make connections between the numeric, graphical, and algebraic representations of polynomial functions;
- identify and describe some key features of the graphs of rational functions, and represent rational functions graphically;
- solve problems involving polynomial and simple rational equations graphically and algebraically;
- demonstrate an understanding of solving polynomial and simple rational inequalities.

Characteristics of Functions:

By the end of this course, students will:

- demonstrate an understanding of average and instantaneous rate of change, and determine, numerically and graphically, and interpret the average rate of change of a function over a given interval and the instantaneous rate of change of a function at a given point;
- determine functions that result from the addition, subtraction, multiplication, and division of two functions and from the composition of two functions, describe some properties of the resulting functions, and solve related problems;
- compare the characteristics of functions, solve problems by modelling and reasoning with functions, including problems with solutions that are not accessible by standard algebraic techniques.

COURSE CONTENT AND EVALUATION

Unit	Unit Description	Specific Expectation Reference	Evaluation:	Hours
Unit 1	**Polynomial Functions:**		10%	16
	1.1 Power Functions	C1.3, 1.4		
	1.2 Characteristics of Polynomial Functions	C1.2, 1.2		
	1.3 Equations and Graphs of Polynomial Functions	C1.5, 1.7, 3.3		
	1.4 Transformations	C1.6, 1.9		
	1.5 Slopes of Secants and Average Rate of Change	D1.1, 1.2, 1.3, 1.4, 1.7. 1.9		
	1.6 Slopes of Tangents and Instantaneous Rate of Change	D1.5, 1.6, 1.8, 1.9		
Unit 2	**Polynomial Equations and Inequalities:**		10%	16
	2.1 The Remainder Theorem	C3.1, 3.2		
	2.2 The Factor Theorem	C3.2		
	2.3 Polynomial Equations	C3.3, 3.4		
	2.4 Families of Polynomial Functions	C1.8		
	2.5 Solve Inequalities Using Technologies	C4.2		
	2.6 Solve Factorable Polynomial Inequalities Algebraically	C4.2, 4.3		
Unit 3	**Rational Functions:**		5%	10
	3.1 Reciprocal of a Linear Function	C2.2		
	3.2 Reciprocal of a Quadratic Function $\frac{ax+b}{cx+d}$	C2.1		
	3.3 Rational Function of the form f(x) =	C1.3, 3.5		
	3.4 Solve Rational Equations ane Inequalities	C3.6, 4.1		
	3.5 Making Connections with Rational Functions and Equations	C3.7		
Unit 4	**Trigonometry:**		10%	15
	4.1 Radian Measure	B1.1, 1.2		
	4.2 Trigonometric Ratios and Special Angles	B1.3, 1.4		
	4.3 Equivalent Trigonometric Expressions	B3.1		
	4.4 Compound Angle Formulas	B3.2		
	4.5 Prove Trigonometric Identities	B3.3		

Unit 5	**Trigonometric Functions:**		10%	15
	5.1 Graphs of Sine, Cosine and Tangent Functions	B2.1, 2.2		
	5.2 Graphs of Reciprocal Trigonometric Functions	B2.3,		
	5.3 Sinusoidal Functions of the Form f(x) = a sin [k(x-d)] +c and f(x) = a cos [k(x-d)] +c	B2.4, 2.5, 2.6		
	5.4 Solve Trigonometric Equations	B3.4		
	5.5 Making Connections and Instantaneous Rate of Change	D1.4		
Unit 6	**Exponential and Logarithmic Functions:**		10%	15
	6.1 The Exponential Function and its Inverse	A1.3, 2.2		
	6.2 Logarithms	A1.1, 1.2		
	6.3 Transformations of Logarithmic Functions	A2.1, 2.3		
	6.4 Power Laws of Logarithms			
	6.5 Making connections: Logarithmic Scales in the Physical Sciences	A2.4		
Unit 7	**Solving Exponential and Logarithmic Equations:**		10%	15
	7.1 Equivalent Forms of Exponential Equations	A3.2		
	7.2 Techniques for Solving Exponential Equations	A3.2, 3.5		
	7.3 Product and Quotient Laws of Logarithms	A1.4, 3.4		
	7.4 Techniques for Solving Logarithmic Equations	A3.1, 3.3		
	7.5 Making connections: Mathematical Modelling with Exponential and Logarithmic Equations	A3.4		
Unit 8	**Combining Functions:**		5%	8
	8.1 Sums and Differences of Functions	D2.1, 2.8		
	8.2 Products and Quotients of Functions	D2.3		
	8.3 Composite Functions	D2.4, 2.5, 2.7		
	8.4 Making Connections: Modeling with Combined Functions	D2.2, 2.6		
	Term Work Total		**70%**	**110**
Final Evaluation	**Final Examination**		**30%**	
	Final Mark		**100%**	

Knowledge/ Understanding	Thinking/ Inquiry	Communication	Application/Making Connections
(30%)	**(30%)**	**(20%)**	**(20%)**
• Quizzes • Worksheets • Assignments • Note making	• Tests • Assignments • Gizmos	• Assignments • Gizmos	• Investigations • Using technology

ASSESSMENT AND EVALUATION

Evaluation and Reporting of Student Achievement

Student achievement is communicated formally to students and parents twice per semester by means of the Provincial Report Card, Grades 9–12. The report card provides a record of the student's achievement of the curriculum expectations in every course, at particular points in the school year or semester, in the form of a percentage grade. The percentage grade represents the quality of the student's overall achievement of the expectations for the course which are described in the achievement chart on pages 28-29 of The Ontario Curriculum Grades 11 and 12, Mathematics, 2007, http://www.edu.gov.on.ca/eng/curriculum/secondary/math1112currb.pdf.

Learning Skills will also be assessed and reported on the Provincial Report Card, Grades 9-12. The quality of the learning skills demonstrated by a student are recorded in six categories – Responsibility, Organization, Independent Work, Collaboration, Initiative, Self-Regulation – and are assessed throughout the semester using a four-point scale (E-Excellent, G-Good, S-Satisfactory, N-Needs Improvement), and using the document Growing Success: Assessment, Evaluation and Reporting in Ontario Schools, 2010 (page 11), as a guide (http://www.edu.gov.on.ca/eng/policyfunding/growSuccess.pdf).

Assessment for Learning will be used as a process for seeking and interpreting evidence for use by learners and their teachers to decide where the learners are in their learning, where they need to go, and how best to get there. Teachers will use diagnostic assessment before instruction and formative assessment will occur frequently and in an ongoing manner to monitor students' progress. Observation and conversation will be used to determine the needs of individual student learning.

Assessment as Learning will focus on the explicit fostering of student's capacity over time to be their own best assessors, but teachers need to start by presenting and modeling external, structured opportunities for students to assess themselves. Formative assessment be used by students to monitor their own and their peers' progress.

Assessment of Learning will be used as the assessment that becomes public and results in statements or symbols about how well students are learning. Summative assessment will be used by the teacher to summarize learning at a given point in time. (Ref: page 31 of Growing Success, or page 8 of this document)

A final grade is recorded for every course, and a credit is granted and recorded for every course in which the student's grade is 50% or higher. The final grade for each course in Grades 9–12 will be determined as follows:

• Seventy per cent of the grade will be based on evaluations conducted throughout the course. This portion of the grade should reflect the student's most consistent level of achievement throughout the course, although special consideration should be given to more recent evidence of achievement. Please see the following page for an explanation of how course work marks will be obtained.

• Thirty per cent of the grade will be based on a final evaluation in the form of an examination, performance task and/or other method of evaluation suitable to the course content and administered towards the end of the course. This final evaluation will include a Culminating Activity 10% and a Final Exam 20%.

A student's achievement of the overall curriculum expectations will be evaluated in accordance with the achievement charts in the provincial curriculum and will be reported using percentage marks. It is expected that both mathematical calculations and professional judgement will inform the determination of percentage marks.

TEACHING AND LEARNING STRATEGIES

Since the over-riding aim of this course is to help students use the language of mathematics skillfully, confidently and flexibly, a wide variety of instructional strategies are used to provide learning opportunities to accommodate a variety of learning styles, interests and ability levels. Seven mathematical processes will form the heart of the teaching and learning strategies used.

- *Communicating:* To improve student success there will be several opportunities for students to share their understanding both in oral as well as written form.
- *Problem solving:* Scaffolding of knowledge, detecting patterns, making and justifying conjectures, guiding students as they apply their chosen strategy, directing students to use multiple strategies to solve the same problem, when appropriate, recognizing, encouraging, and applauding perseverance, discussing the relative merits of different strategies for specific types of problems.
- *Reasoning and proving:* Asking questions that get students to hypothesize, providing students with one or more numerical examples that parallel these with the generalization and describing their thinking in more detail.
- *Reflecting:* Modeling the reflective process, asking students how they know.
- *Selecting Tools and Computational Strategies:* Modeling the use of tools and having students use technology to help solve problems.
- *Connecting:* Activating prior knowledge when introducing a new concept in order to make a smooth connection between previous learning and new concepts and introducing skills in

context to make connections between particular manipulations and problems that require them.

- *Representing:* Modeling various ways to demonstrate understanding, posing questions that require students to use different representations as they are working at each level of conceptual development - concrete, visual or symbolic, allowing individual students the time they need to solidify their understanding at each conceptual stage.

CONSIDERATIONS FOR PROGRAM PLANNING

The planning and administering of this course is based on the premise that all students can be successful language learners. The teacher will provide quality instruction that respects students' strengths and address their learning needs, using assessment information to plan instruction. Accommodation for IEP students is not applicable at Niagara Academy, but the teacher will make appropriate adaptions for English Language Learners, such as the following:

- modification of some or all of the course expectations, based on the student's level of English proficiency;
- use of a variety of instructional strategies (e.g. extensive use of visual cues, manipulatives, pictures, diagrams, graphic organizers; attention to clarity of instructions; modelling of preferred ways of working in mathematics; previewing of textbooks; pre-teaching of key specialized vocabulary; encouragement of peer tutoring and class discussion; strategic use of students' first languages);
- use of a variety of learning resources (e.g., visual material, simplified text, bilingual dictionaries, culturally diverse materials);
- use of assessment accommodations (e.g., granting of extra time; use of alternative forms of assessment, such as oral interviews, learning logs, or portfolios; simplification of language used in problems and instructions).

Information and communications technology will be used throughout the course where it is appropriate. The program will also include opportunities for students to apply their skills to work-related situations, to explore educational and career options, and to become self-directed learners.

Teachers will model safe practices at all time and communicate safety requirements to students in accordance with the school, the Ministry of Education policies and Ministry of Labour regulations. This is particularly important in the case of off-site excursions. Teachers will also adhere to the policies of the First Nation, Métis and Inuit Education policy framework should the class include students from these communities.

RESOURCES

Advanced Functions 12, McGraw-Hill Ryerson, 2008, ISBN 0-07-012710-7

Functions and Relations 11, Addison-Wesley, 2002, ISBN 0-201-72657-2 MCR3U-C (ILC) The Ontario Educational Communications Authority, 2012